I solved Loschmidt's Paradox
aka
Undermine the paradigm

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A guy named Johann Loschmidt criticized a guy named Ludwig Boltzmann in 1876 (at least that's when it was published). Today this is known as Loschmidt's Paradox. Many science fans learn of the second law of thermodynamics and the arrow of time: entropy increases in the future but not the past. Or $\frac{dS}{dt} > 0$. At the same time, fundamental physical theories like quantum electrodynamics (accurate to ten parts in a billion) have solutions which really don't change when t is transformed to $-t$. Read on, kid.

1 Introduction

Loschmidt's Paradox examines the nature of time. [1] The paradox says something like this, "It should not be possible to have an irreversible process (entropy only increases in the future, $\frac{dS}{dt} > 0$) from time-symmetric dynamics." I am going to teach you how to solve Loschmidt's Paradox. You have three steps: learn what time symmetry is, learn the second law of thermodynamics, and learn how to perform the mathematics. How will you solve Loschmidt's Paradox? You will understand that time-asymmetric dynamics, that is dynamics with equations of motion which are time-reversal invariant, are a red herring. You will undermine the paradigm.

1.1 T-symmetry from college

All of physics is "reversible" at the fundamental level in the sense that you can put a *minus* sign in front of the variable t for time in the equations and get the same answer. This is called time symmetry or T-symmetry.[2]

If you know Newtonian physics and what a derivative is, you can do this. Remember that *velocity* is $v = \frac{dx}{dt}$ and *acceleration* is $a = \frac{d^2x}{dt^2}$. In college mechanics you learn that this equation predicts the motion of a cannonball.

$$x_f = x_i + v_0 t + \frac{1}{2}at^2 \tag{1}$$

If you put a minus sign in front of the t's, Equation 1 remains *unchanged*.[3] You can say it is invariant under "time reversal" or obeys time symmetry.[4]

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[2] Technically, to make a statement without flaws, experiment tells us the laws of nature are only invariant under a combined symmetry called CPT. But here we only focus on time. There is nothing lost in this discussion under this approach. I already solved the full symmetry. Also, forget about kaons for now. [2] It's alright.

[3] i) $v \to \frac{dx}{-dt} = -\frac{dx}{dt} = -v$ and $a \to \frac{d}{-dt}\frac{dx}{-dt} = \frac{d^x}{dt^2} = a$

ii) $x_f = x_i + (-v_0)(-t) + \frac{1}{2}(+a)(-t)^2 = x_i + v_0 t + \frac{1}{2}at^2$

[4] The truth is the same applies in quantum mechanics and Einstein's theory of general relativity.

1.2 The arrow of time

In thermodynamics, there is an arrow of time associated with entropy because it only increases in the future and not the past. [3] This is the common "you can't put toothpaste back in the tube" idea. Equation 1 above won't tell you if time moves forward or backward because the minus sign doesn't matter. But the toothpaste will not re-enter the tube. Many authorities, among them Schrödinger, have thus wondered as Loschmidt had about the disparity. [4]

2 How to solve Loschmidt's Paradox

2.1 Another universe

Picture yourself in another universe where physics does not obey T-symmetry. You will learn physics. You will make predictions like the path of a cannonball. You will learn about thermodynamics and experience the arrow of time. But what is Loschmidt left with to say? Entropy increases in the future in that universe and I can prove it by simulating it. Don't worry about the details. I just order my computer to bounce balls of energy around a cube.

The movement is discrete and the equations of motion are $\Psi_{t+dt} = U_t\Psi_t$ with the simple state $\Psi_t = \begin{pmatrix} x_t & v_t \end{pmatrix}^T$.[5] T-symmetry is preserved if $U_{t+dt}RU_t = R$ for all t, where R is a transformation that inverts velocity.[6]

At this point, I leave some of the mathematics to you. The full proof is online.[7] It would take time to grapple even if interested in the challenge. You must be comfortable with operators, matrices, summations, the Dirac delta function, and generalization. When I first solved this,

$$U_{t+dt}RU_t\Psi_t = \begin{pmatrix} 1 & (1-\gamma P\theta III)_{t+dt} \\ 0 & (1-2P\theta III)_{t+dt} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & (1-\gamma P\theta III)_t \\ 0 & (1-2P\theta III)_t \end{pmatrix} \begin{pmatrix} x_t \\ v_t \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} 1 & (1-\gamma P\theta III)_t - (1-2P\theta III)_t(1-\gamma P\theta III)_{t+dt} \\ 0 & -(1-2P\theta III)_t(1-2P\theta III)_{t+dt} \end{pmatrix} \begin{pmatrix} x_t \\ v_t \end{pmatrix}$$

$$= \begin{pmatrix} x_t \\ -v_t \end{pmatrix} \text{ if } \gamma = 1 \text{ and a bounce occurs @ } t \tag{3}$$

$$= \begin{pmatrix} x_t \\ v_t \end{pmatrix} \text{ if } \gamma = 2 \text{ and a bounce occurs @ } t \tag{4}$$

Try it yourself. How does this solve Loschmidt's Paradox? Well... basically... $\frac{dS}{dt} > 0$ whether $U_{t+1}RU_t = R$ or not. Did you have questions?[8,9] Believe it or not, this is **not** in the scientific literature. At least not to my knowledge. But the interested reader may find a useless paradigm approach in [5].

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[5] Discrete because it *is* a simulation. The variables x and v represent position and velocity.

[6] In a "normal" environment, one should be able to step forward (U_t), turn around (R), and step again (U_{t+1}) and end up in the same position facing the opposite direction. Consider this a test of T-symmetry.

[7] Broken time-reversal invariance in deterministic spin systems

[8] Have imagination? Picture all the particles in one corner of the cube. Later, they're not. T-symmetry has nothing to do with it. ;-).

[9] The thermodynamical consequences of breaking T-symmetry tell us something about our own universe. See the Figures in "Broken time-reversal invariance in deterministic spin systems" for examples. This is by far the most interesting aspect of this work. T-symmetry assures thermal equilibrium and maximizes entropy. I salivate at teaching you the consequences for the black hole information paradox.

Acknowledgments

Buy me something.[10]

References

[1] J. Loschmidt, Sitzungsber. Kais. Akad. Wiss. Wien, Math. Naturwiss. Classe 73, 128142 (1876)

[2] A. Angelopoulos et al. The European Physical Journal C, September 2001, Volume 22, Issue 1, pp 55-79

[3] Statistical Mechanics, Entropy, Order Parameters, and Complexity by James P. Sethna. Page 80.

[4] Lebowitz Macroscopic laws, microscopic dynamics Boltzmann entropy, Physica A 194.

[5] Holian,Hoover,Posch Resolution of Loschmidt's Paradox- The Origin of Irreversible Behavior in Reversible Atomistic Dynamics.

These textbooks serve well for history and context from a technical perspective.

[6] Zeh HD. The physical basis of the direction of time. 4th ed. New York; Berlin; Springer; 2001.

[7] Davies, P. C. W. The Physics of Time Asymmetry. Berkeley: University of California Press, 1974.

[8] Davies, P. C. W. About Time: Einstein's Unfinished Revolution. New York: Simon & Schuster, 1995.

[9] Sachs, Robert Green. The Physics of Time Reversal. Chicago: University of Chicago Press, 1987.

[10] Skalsey, Mark, and American Institute of Physics. "Time Reversal: The Arthur Rich Memorial Symposium, Ann Arbor, MI 1991".

[11] Halliwell, J. J., J. Perez-Mercader, and W.H. Zurek. Physical Origins of Time Asymmetry. Eds., Cambridge University Press, 1994.

These journal articles provide further context. Must excuse format. Set 2 has more relevance than Set 1.

Set 1.

[12] Bratschi,Huber,Searles Non-Hamiltonian molecular dynamics implementation of the Gibbs ensemble method. II. Molecular liquid-vapor results for carbon dioxide.

[13] Cover Which Processes Satisfy the Second Law.

[14] Ezra On the statistical mechanics of non-Hamiltonian systems- the generalized Liouville equation, entropy, and time-dependent metrics.

[15] Heringa,Shinkai,Blote,Hoogland,Zia Bistability in an Ising model with non-Hamiltonian dynamics.

[16] Hoover,Posch,Aoki,Kusnezov Remarks on non-Hamiltonian statistical mechanics- Lyapunov exponents and phase-space dimensionality loss.

[10] https://www.google.com/search?q=hoplology

[17] Leonard,Roelly,Zambrini On the time symmetry of some stochastic processes.

[18] Lin Time Symmetry and Thermodynamics.

[19] Marro,Fernandez,Gonzalez-Miranda,Puma Ising critical behavior of a non-Hamiltonian lattice system.

[20] Masia An Introduction to Advanced Molecular Dynamics Techniques (lecture slides).

[21] Neal MCMC using Hamiltonian dynamics.

[22] Popescu,Popescu,Dumitru Some aspects concerning non-Hamiltonians systems.

[23] Prigogine,George,Henin Dynamics of Systems with Large Number of Degrees of Freedom and Generalized Transformation Theory.

[24] Sergi,Ezra ALGORITHMS FOR NON-HAMILTONIAN DYNAMICS.

[25] Tuckerman,Liu,Ciccotti,Martyna Non-Hamiltonian molecular dynamics- Generalizing Hamiltonian phase space principles to non-Hamiltonian systems.

[26] Zhukov,Cao Kinetic theory of non-hamiltonian statistical ensembles.

Set 2.

[27] Bohm,Gadella,Kielanowski Time Asymmetric Quantum Mechanics.

[28] Brown,Myrvold,Uffink Boltzmanns H-theorem, its discontents, and the birth of statistical mechanics.

[29] Callens,De Roeck,Jacobs,Maes,Netocn? Quantum entropy production as a measure of irreversibility.

[30] Crooks On thermodynamic and microscopic reversibility.

[31] Esposito,Lindenberg,Van den Broeck Entropy production as correlation between system and reservoir.

[32] Evans Search google for pdf: “selectnewsreportsFT.pdf”.

[33] Evans,Searles Equilibrium microstates which generate second law violating steady states.

[34] George,Prigogine,Rosenfeld The Macroscopic Level of Quantum Mechanics.

[35] Henley Time Reversal Symmetry.

[36] Hinds Testing time reversal symmetry using molecules.

[37] R. Illner and H. Neunzert The concept of irreversibility in the kinetic theory of gases, Transp. Th. Stat. Phys. 16:89-112, (1987)

[38] Lebowitz Boltzmann’s entropy and time’s arrow.

[39] Lebowitz Microscopic reversibility and macroscopic behavior- Physical explanatoins and mathematical derivations.

[40] Lebowitz Boltzmanns entropy, times arrow, Phys. today.

[41] Lebowitz Microscopic origins of irreversible macroscopic behavior, PhysicaA.

[42] Lucia Irreversibility, entropy and incomplete information.

[43] Page No Time Asymmetry from Quantum Mechanics.

[44] Prez-Madrid Generalized Gibbs Entropy, Irreversibility and Stationary States.

[45] Prez-Madrid Gibbs entropy and irreversibility.

[46] Prez-Madrid Quantum theory of irreversibility.

[47] Poetz Theoretical Study of Quantum Systems Interacting with a Phonon Bath.

[48] Prosper Temperature fluctuations in a heat bath.

[49] Ramshaw Entropy ambiguity in a system in equilibrium with a finite heat bath.

[50] Rhim,Pines,Waugh Time-Reversal Experiments in Dipolar-Coupled Spin Systems.

[51] Rhim,Pines,Waugh VIOLATION OF THE SPIN- TEMPERATURE HYPOTHESIS.

[52] CPLEAR Collaboration T -violation and CPT -invariance measurements in the CPLEAR experiment- a detailed description of the analysis of neutral-kaon decays to epineutrino.

[53] Wang,Sevick,Mittag,Searles,Evans Experimental Demonstration of Violations of the Second Law of Thermodynamics for Small Systems and Short Time Scales.

[54] Zeh QUANTUM THEORY AND TIME ASYMMETRY.

[55] Uffink Bluff your way in the Second Law of Thermodynamics.

[56] Lieb,Yngvason The Physics and Mathematics of the Second Law of Thermodynamics.